



ISMIR Tutorial

Daejeon, Korea, September 21, 2025



Differentiable Alignment Techniques for Music Processing: Techniques and Applications

Part 1: Introduction to Alignment Techniques

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International Audio Laboratories Erlangen {meinard.mueller, johannes.zeitler}@audiolabs-erlangen.de





Overview

Part 0: Overview

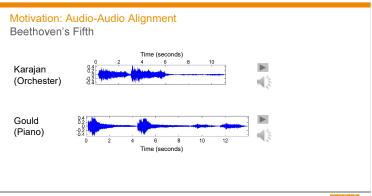
Part 1: Introduction to Alignment Techniques

Coffee Break

Part 2: Theoretical Foundations & Implementation

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Motivation: Audio-Audio Alignment

Beethoven's Fifth

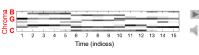
Karajan

(Orchester)



Time-chroma representations

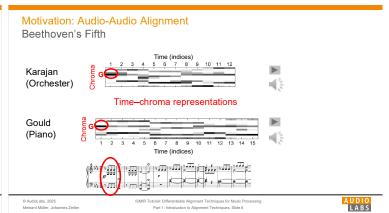
Gould (Piano)

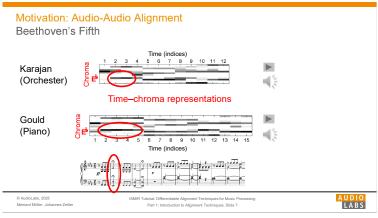


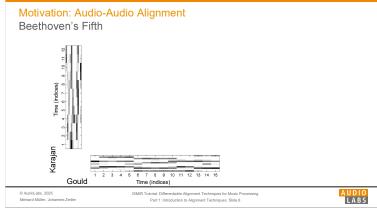
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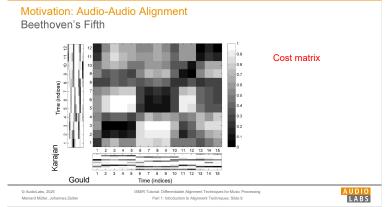
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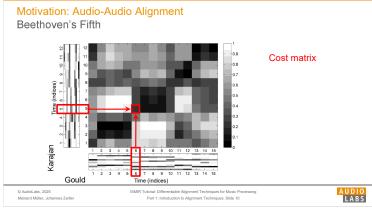
Motivation: Audio-Audio Alignment Beethoven's Fifth Karajan (Orchester) Time-chroma representations Gould (Piano) Gould (Piano) Gould (Piano) For Items (Indices) Time (Indices)

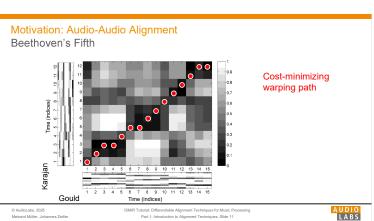


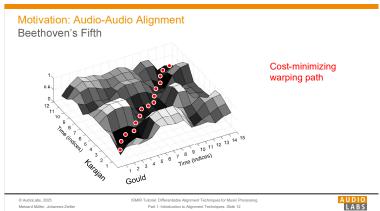












Motivation: Audio-Audio Alignment

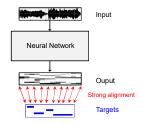
Beethoven's Fifth

Karajan (Orchester) Gould (Piano)

Cost-minimizing warping path

→ Strong alignment

Feature Learning



- Task: Learn audio features using a neural network
- Loss: Binary cross-entropy
 - framewise loss
 - requires strongly aligned targets
 - hard to obtain

Time (indices)



Alignment matrix

 $\mathcal{A}_{N,M} \subset \{0,1\}^{N \times M}$

Alignment matrix

 $\mathcal{A}_{N,M} \subset \{0,1\}^{N \times M}$

Set of all possible alignment matrices

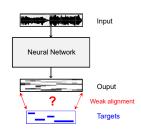
 $A \in \{0,1\}^{N \times M}$

Set of all possible alignment matrices

 $A \in \left\{0,1\right\}^{N \times M}$

LABS

Feature Learning



- Task: Learn audio features using a neural network
- Loss: Binary cross-entropy
- framewise loss
- requires strongly aligned targets
- hard to obtain
- Alignment as part of loss function
 - requires only weakly aligned targets
 - needs to be differentiable
- Problem: DTW is not differentiable → Soft DTW

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Dynamic Time Warping (DTW)

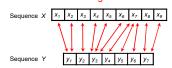
$$X := (x_1, x_2, \dots, x_N)$$

$$Y:=(y_1,y_2,\ldots,y_M)$$

$$x_n, y_m \in \mathcal{F}$$
, $n \in [1:N]$, $m \in [1:M]$

$$\mathcal{F}$$
 = Feature space

Alignment





Dynamic Time Warping (DTW)

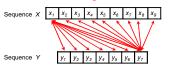
$$X:=(x_1,x_2,\ldots,x_N)$$

$$Y:=(y_1,y_2,\ldots,y_M)$$

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= Feature space

Alignment



Alignment matrix $A \in \{0,1\}^{N \times M}$

Set of all possible alignment matrices $\mathcal{A}_{N,M} \subset \{0,1\}^{N \times M}$



Dynamic Time Warping (DTW)

$$X:=(x_1,x_2,\ldots,x_N)$$

$$Y:=(y_1,y_2,\ldots,y_M)$$

$$x_n,y_m\in\mathcal{F}$$
, $n\in[1:N]$, $m\in[1:M]$

 \mathcal{F} = Feature space

 $c: \mathcal{F} \times \mathcal{F} \to \mathbb{R}_{\geq 0}$ Cost measure:

 $C \in \mathbb{R}^{N \times M}$ with $C(n, m) := c(x_n, y_m)$ Cost matrix:

Cost of alignment: $\langle A, C \rangle$

DTW cost: $\mathrm{DTW}(C) = \min \left(\left\{ \left\langle A, C \right\rangle \ | \ A \in \mathcal{A}_{N,M} \right\} \right)$ Optimal alignment: $A^* = \operatorname{argmin}\left(\{\langle A,C \rangle \mid A \in \mathcal{A}_{N,M}\}\right)$

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Dynamic Time Warping (DTW)

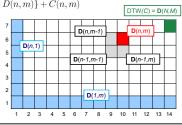
DTW cost: $\mathrm{DTW}(C) = \min \left(\left\{ \left\langle A, C \right\rangle \ \middle| \ A \in \mathcal{A}_{N,M} \right\} \right)$

■ Efficient computation via Bellman's recursion in O(NM)

$$D(n,m) = \min\{D(n-1,m), D(n,m-1), D(n,m)\} + C(n,m)$$

for n>1 and m>1 and suitable initialization.

$$DTW(C) = D(N, M)$$



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Dynamic Time Warping (DTW)

 $\mathrm{DTW} \operatorname{cost:} \qquad \mathrm{DTW}(C) = \min \left(\left\{ \left\langle A, C \right\rangle \ \middle| \ A \in \mathcal{A}_{N,M} \right\} \right)$

■ Efficient computation via Bellman's recursion in O(NM)

$$D(n,m) = \min\{D(n-1,m), D(n,m-1), D(n,m)\} + C(n,m)$$

for n>1 and m>1 and suitable initialization.

$$DTW(C) = D(N, M)$$

- Problem: DTW(C) is not differentiable with regard to C
- Idea: Replace min-function by a smooth version

$$\min^{\gamma} (S) = -\gamma \log \sum_{s \in S} \exp(-s/\gamma)$$

for set $\, \mathcal{S} \subset \mathbb{R} \,$ and temperature parameter $\, \gamma \in \mathbb{R} \,$

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Soft Dynamic Time Warping (SDTW)

 $\mathrm{SDTW} \ \mathrm{cost:} \quad \mathrm{SDTW}^{\gamma}(C) = \min^{\gamma} \left(\left\{ \left\langle A, C \right\rangle \ | \ A \in \mathcal{A}_{N,M} \right\} \right)$

• Efficient computation via Bellman's recursion in O(NM) still works:

$$D^{\gamma}(n,m)=\min^{\gamma}\{D^{\gamma}(n-1,m),D^{\gamma}(n,m-1),D^{\gamma}(n,m)\}+C(n,m)$$

for n>1 and m>1 and suitable initialization.

 $SDTW^{\gamma}(C) = D^{\gamma}(N, M)$

 $\qquad \text{Limit case:} \quad \mathrm{SDTW}^{\gamma}(C) \xrightarrow{\gamma \to 0} \mathrm{DTW}(C)$

■ SDTW(C) is differentiable with regard to C

- Questions:
 - How does the gradient look like?
 - Can it be computed efficiently?
 - How does SDTW generalize the alignment concept?

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Soft Dynamic Time Warping (SDTW)

SDTW cost: $SDTW^{\gamma}(C) = \min^{\gamma} \left(\{ \langle A, C \rangle \mid A \in \mathcal{A}_{N,M} \} \right)$

 ${\color{red} \bullet}$ Define $\,p^{\gamma}(C)\,$ as the following "probability" distribution over ${\mathcal A}_{N,M}$:

$$p^{\gamma}(C)_{A} = \frac{\exp\left(-\langle A,C\rangle/\gamma\right)}{\sum_{A'\in\mathcal{A}_{N,M}} \exp\left(-\langle A',C\rangle/\gamma\right)} \qquad \quad \text{for } A\in\mathcal{A}_{N,M}$$

- The expected alignment with respect to $\,p^{\gamma}(C)\,$ is given by:

$$E^{\gamma}(C) = \sum\nolimits_{A \in \mathcal{A}_{NM}} p^{\gamma}(C)_{A} A \quad \in \mathbb{R}^{N \times M}$$

The gradient is given by:

$$\nabla_C \mathrm{SDTW}^{\gamma}(C) = E^{\gamma}(C)$$

• The gradient can be computed efficiently in O(NM) via a recursive algorithm.

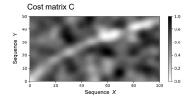
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Soft-DTW Cuturi, Blondel: Soft-DTW: A Differentiable Loss Function for Time-Series. ICML, 2017

Soft Dynamic Time Warping (SDTW)

 $\text{Expected alignment}: \quad E^{\gamma}(C) = \sum\nolimits_{A \in \mathcal{A}_{N,M}} p^{\gamma}(C)_A A \quad \in \mathbb{R}^{N \times M}$

- Can be interpreted as a smoothed version of an alignment
- Degree of smoothing depends on temperature parameter γ



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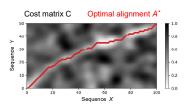
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Soft Dynamic Time Warping (SDTW)

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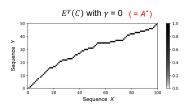
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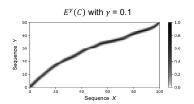
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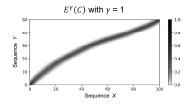
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Soft Dynamic Time Warping (SDTW)

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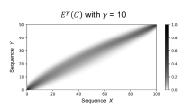
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Soft Dynamic Time Warping (SDTW)

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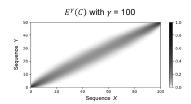
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Soft Dynamic Time Warping (SDTW)

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Soft Dynamic Time Warping (SDTW)

Conclusions

- Direct generalization of DTW (replacing min by smooth variant)
- Gradient is given by expected alignment
- Fast forward algorithm: O(NM)
- Fast gradient computation: O(NM)
- SDTW yields a (typically) poor lower bound for DTW
- Can be used as loss function to learn from weakly aligned sequences

Soft Dynamic Time Warping (SDTW)

References

- Marco Cuturi, Mathieu Blondel: Soft-DTW: A Differentiable Loss Function for Time-Series. ICML, pages 894–903, 2017.
- Arthur Mensch, Mathieu Blondel: Differentiable Dynamic Programming for Structured Prediction and Attention, ICML, 2018.
- Michael Krause, Christof Weiß, Meinard Müller: Soft Dynamic Time Warping for Multi-Pitch Estimation and Beyond. IEEE ICASSP, 2023.

Thanks: Michale Krause (Ph.D. 2023) Johannes Zeitler (Ph.D.)



LABS

CTC Loss: Introduction

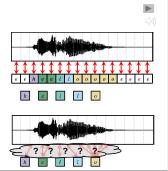
- Connectionist Temporal Classification (CTC)
- Graves, Fernández, Gomez, and Schmidhuber: Connectionist Temporal Classification: Labelling Unsegmented Sequence Data with Recurrent Neural Networks. ICML, 2006.
- Temporal Classification: Labelling unsegmented data sequences
- Connectionist: Refers to the use of deep learning



CTC Loss: Introduction

Training data in speech recognition

- Strongly aligned training data
 - Character annotations (labels) for each time step
 - Can be used for training in a standard classification
 - Tedious to annotate
- Weakly aligned training data
 - Globally corresponding character sequence without local alignment
 - Cannot be used for training in a standard classification setup
 - Easier to annotate
- Aim of CTC: Employ only weakly aligned data for



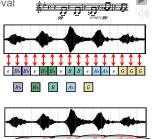
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CTC Loss: Introduction

Training data in theme-based music retrieval

Strongly aligned training data

- Character annotations (labels) for each time step
- Can be used for training in a standard classification
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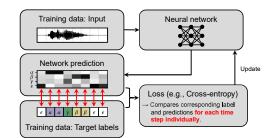
Bb Bb B Ab G

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CTC Loss: Introduction

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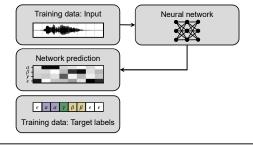
Standard deep learning setup: Strongly aligned training data



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CTC Loss: Introduction

Standard deep learning setup: Strongly aligned training data



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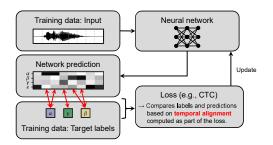
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CTC Loss: Introduction

Non-standard deep learning setup: Weakly aligned training data



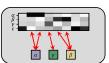
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LABS

CTC Loss: Introduction

Alignment Representations

"Arrow" representation



"Point" representation



"Unfolded" representation





CTC Loss: Introduction

- Alphabet $\mathbb{A} = \{\alpha, \beta, \gamma\}$
- Label sequence Y = (α, γ, β)
- αγβ
- Network output
- $f_{\theta}(X) =$



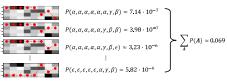
- Alignment A is "expansion" of Y to length of $f_{\theta}(X)$ (possibly consecutive duplicates and blank symbols ϵ)

CTC Loss: Introduction

- Alphabet A = {α, β, γ}
- Label sequence Y = (α, γ, β)
- Naive idea: "Hard" alignment (Related: Viterbi decoding)
- $P(\epsilon, \alpha, \alpha, \gamma, \beta, \beta, \epsilon, \epsilon) \approx 0.015$
- Not suitable for gradient-descent-based training (not differentiable)

αγβ

Therefore: "Soft" alignment (Related: Forward algorithm)



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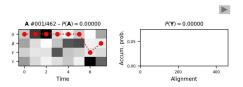


CTC Loss: Introduction

- Alphabet $\mathbb{A} = \{\alpha, \beta, \gamma\}$
- Label sequence Y = (α, γ, β)
- Naive idea: "Hard" alignment (Related: Viterbi decoding)



- Not suitable for gradient-descent-based training (not differentiable)
- Therefore: "Soft" alignment (Related: Forward algorithm)



Theme-Based Audio Retrieval

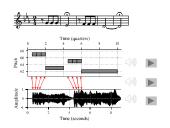
Barlow & Morgenstern (1949): A Dictionary of Musical Themes





- 2067 themes by 54 different composers
- Recordings (1126 recordings, ~ 120 hours)
- Theme occurences (~ 5 hours)

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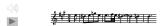
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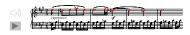
Theme-Based Audio Retrieval

Monophony-Polyphony Challenge

Monophonic symbolic musical theme



Audio recording of polyphonic music



Chromagram

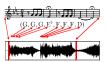


Goal: Compute "enhanced" chromagram from polyphonic audio recording that better matches the symbolic monophonic theme

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Theme-Based Audio Retrieval

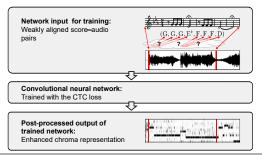
Strongly Aligned Training Data





Theme-Based Audio Retrieval

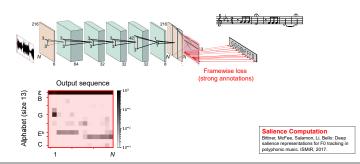
Weakly Aligned Training Data



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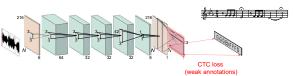
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Theme-Based Audio Retrieval



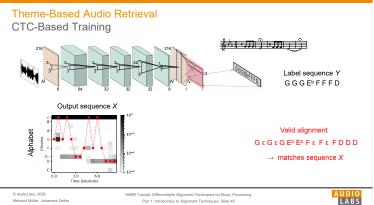
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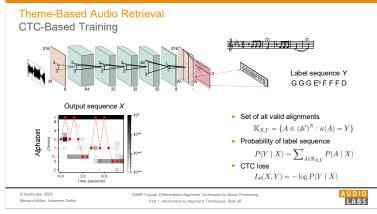
Theme-Based Audio Retrieval

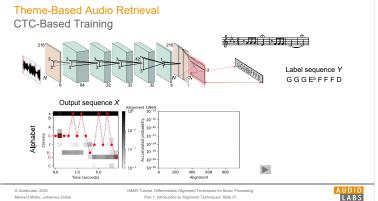


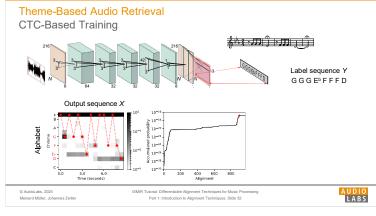
- Idea of CTC loss similar to SDTW
- Theme is given as label sequence over finite alphabet (size 13 including blank symbol)
- Expand label sequence to match audio feature sequence
- CTC loss considers probability over all valid alignments
 → differentiable

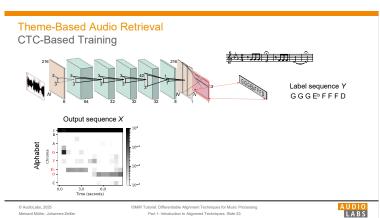
Theme-Based Audio Retrieval CTC-Based Training ارُ الراس ارُ الرحيِّالِيُّ Label sequence Y GGGE^bFFFD Output sequence X

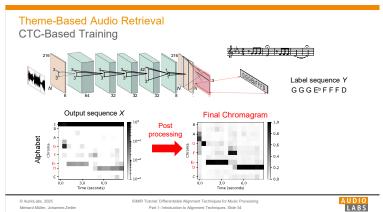








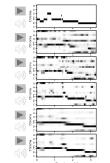




Theme-Based Audio Retrieval **Evaluation Results**



Chroma Variant	Top-1	Top-10
Standard chromagram	0.561	0.723
Enhanced chromagram (baseline)	0.824	0.861
DNN-based chromagram (CTC)	0.867	0.942
DNN-based chromagram (linear scaling)	0.829	0.914
DANIE II (I E O	0.000	0.000



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Theme-Based Audio Retrieval

References

- R. Bittner, B. McFee, J. Salamon, P. Li, and J. Bello: Deep salience representations for F0 tracking in polyphonic music. Proc. ISMIR, pages 63–70, 2017.
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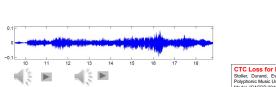
Frank Zalkow (Ph.D. 2021) Stefan Balke (Ph.D. 2018)





Lyrics-Audio Alignment

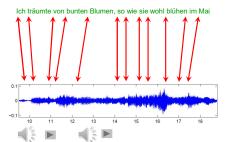
Ich träumte von bunten Blumen, so wie sie wohl blühen im Mai



LABS

AUDIO LABS

Lyrics-Audio Alignment

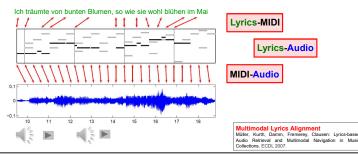


Lyrics-Audio

LABS

Lyrics-Audio Alignment Ich träumte von bunten Blumen, so wie sie wohl blühen im Mai Lyrics-Audio

Lyrics-Audio Alignment



LABS

Datasets

Schubert Winterreise Dataset (SWD)

- Song cycle by Franz Schubert
- 24 songs
- 9 performances (versions)
- Annotations
 - Lvrics
 - Chords
 - Local keys
 - Structure

Weiß et al.: Schubert Winterreise Dataset: A Multimodal Scenario for Music Analysis ACM J. Computing & Cultural Heritage, 2021.

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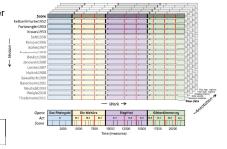
| Cheese (Person) | Cheese (Pe

Datasets

Wagner Ring Dataset (WRD)

- Opera cycle by Richard Wagner
- 4 operas (ca. 22,000 meaures)
- 16 performances (versions)
- Annotations
 - Lvrics
- Measure positions
- Aligned reduced score

Weiß et al.: Wagner Ring Dataset: A Complex Opera Scenario for Music Processing and Computational Musicology, TISMIR 2023.



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Part 1: Introduction to Alignment Techniques Slide 62

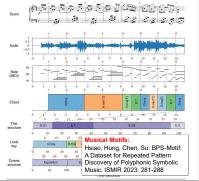


Datasets

Beethoven Piano Sonata Dataset

- Piano Sontats by Beethoven
- 32 first movments
- 11 performances (versions)
- Annotations
 - Notes
 - Measures and beats
 - Chords, local & global keys
 - Musical structures

Zeitler et al.: BPSD: A Coherent Multi-Version Dataset for Analyzing the First Movements of Beethoven's Piano Sonatas. TISMIR 2024.

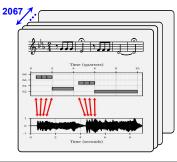


Datasets

Musical Theme Dataset (MTD)

- Western classical music
- Inspired by Barlow & Morgenstern (1948)
- 2067 themes
 - Symbolic encodings
 - Audio excerpts
 - Strong alignments
- ...

Zalkow et al.: MTD: A Multimodal Dataset of Musical Themes for MIR Research. TISMIR 2020.



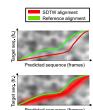
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ISMIR Tutorial: Differentiable Alignment Techniques for Music Process
Part 1: Introduction to Alignment Techniques. Slide 64



Soft Dynamic Time Warping (SDTW) Stabilizing Training

- Standard SDTW often unstable
 - Unstable training in early stagesDegenerate output alignment
- Uhan ann ann an Aran a dhaatan an t
- Hyperparameter adjustment
 - High temperature to smooth alignments
 - Temperature annealing
- Diagonal prior
- Modified step size condition



LABS





Soft Dynamic Time Warping (SDTW)

Representation Learning

- Symmetric application
 - Learn representation of both sequences
 - Needs a contrastive loss term
- Assymmetric application
 - Use fixed (e.g., binary) encoding of target
 - Learn representation of only one sequences
 - No contrastive loss term need
- Simulation of CTC-loss using SDTW possible
- Many DTW variants also possible for SDTW

